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Even for the observed luminosity distance $D_L(z)$ which suggests the existence of the dark energy, we show that the inhomogeneous dust universe solution without the dark energy is possible in general. Future observation of $D_L(z)$ for $1 \lesssim z < 1.7$ may confirm or refute this possibility.

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Recent measurements of the luminosity distance $D_L(z)$ using Type Ia supernovae [1–3] suggest that accurate $D_L(z)$ may be obtained in near future. Especially SNAP [4] will give us the luminosity distance of ~ 2000 Type Ia supernovae with an accuracy of a few % up to $z \sim 1.7$ every year. On the other hand from the observation of the first Doppler peak of the anisotropy of CMB, it is now suggested that the universe is flat [5,6], which may be proved in future by MAP and Planck. Under the assumption of the homogeneity and the isotropy of our universe, these observations suggest that the dark energy is dominant at present. To interpret what the dark energy is [7] many arguments have been done so far. However at present we do not have a firm and reliable theoretical basis to discuss such a small amount of energy scale compared with the Planck one. In short the dark energy under the assumption of homogeneity and the isotropy of our universe is by far the great mystery.

From the observed isotropy of the CMB, if we are not in a special part of our universe, the universe should be homogeneous. However if we are in a special part, the universe might be inhomogeneous although the CMB is isotropic. Such cosmological models have been constructed using spherically symmetric models in which we are near the symmetric center. Some authors have considered such models to interpret the SNIa data for small z [8] as well as up to large z assuming a void structure [9–11] to avoid the dark energy. One may regard such possibilities absurd. However our point of view in this letter is to construct a possible inhomogeneous dust universe derived from the observed $D_L(z)$. If such a model is possible at present, the inhomogeneous universe should be examined more seriously since the dark energy solution is also absurd in the sense that it is ~ 120 orders of magnitude smaller than the Planck scale. In short we suppose one can reduce, somewhat crudely, the question we need to answer to: which is more absurd, the dark energy or the inhomogeneous universe. In the former case there is no reliable theory to examine the problem at present while the latter case can be studied in the frame work of the known theories. We like to point out that not the taste but the future observations will confirm either the dark energy or the inhomogeneous universe.

The analysis of high redshift supernovae gives us the luminosity distance-redshift relation $D_L(z)$ along the observational past null cone up to $z \sim 1$ [1–3]. The data fit well with $D_L(z)$ in the homogeneous and the isotropic universe with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ given by

$$D_L(z) = \frac{1}{H_0}(1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}. \quad (1)$$

In this letter we assume that $D_L(z)$ is given by Eq. (1) with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ for $z \lesssim 1$. This is just for simplicity to make the arguments clearer. We do not claim that $D_L(z)$ with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is confirmed. While $D_L(z)$ for $1 \lesssim z < 1.7$ is not certain even at present and will be obtained in future, for example, by SNAP. Since the scale factor a obeys

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p), \quad (2)$$

$D_L(z)$ with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ means that the present universe is accelerating, while for the dust universe ($p = 0$), a should be decelerating. Therefore one may conclude that the observation is not consistent with the inhomogeneous dust model. However the point is that to determine $D_L(z)$ we are observing Type Ia supernova events occurred at

past time in different spatial positions from us. In the inhomogeneous universe model, the time dependence of a at the different point from us is different so that we may obtain an apparent accelerating universe although the dust universe is decelerating locally.

The line element of a spherically symmetric dust universe is given by

$$ds^2 = -dt^2 + \frac{(R'(t, r))^2}{1 + 2E(r)r^2} dr^2 + R^2(t, r) d\Omega^2, \quad (3)$$

where the prime means the derivative with respect to r . The solution to the Einstein equations is known as the Lemaître-Tolman-Bondi (LTB) spacetime [12–14];

$$\dot{R} = \sqrt{\frac{2M(r)}{R} + 2E(r)r^2} \quad (4)$$

$$4\pi\rho(t, r) = \frac{M'}{R^2 R'}, \quad (5)$$

where the dot means the derivative with respect to t . The solution of Eq. (4) is given by

$$R(t, r) = \frac{M}{\epsilon(r)r^2} \phi(t, r), \quad t - t_B(r) = \xi(t, r) \frac{M}{(\epsilon(r)r^2)^{3/2}}, \quad (6)$$

where

$$\epsilon(r)r^2 = \begin{cases} 2E(r)r^2 & (E(r) > 0) \\ 1 & (E(r) = 0) \\ -2E(r)r^2 & (E(r) < 0) \end{cases}, \quad (7)$$

and

$$\phi = \begin{cases} \cosh \eta - 1 \\ \frac{\eta^2}{2} \\ 1 - \cos \eta \end{cases}, \quad \xi = \begin{cases} \sinh \eta - \eta & (E(r) > 0) \\ \frac{\eta^3}{6} & (E(r) = 0) \\ \eta - \sin \eta & (E(r) < 0) \end{cases}. \quad (8)$$

In the general solutions of the LTB models, there are three arbitrary functions $M(r)$, $E(r)$ and $t_B(r)$. $M(r)$ is regarded as the gravitational mass function and we can set $M(r) = M_0 r^3$ redefining r . $t_B(r)$ corresponds to the local Big-Bang time. $E(r)$ determines the local curvature radius or the local specific energy. The functions $t_B(r)$ and $E(r)$ should be chosen to reproduce the observed $D_L(z)$. This means that we have only one constraint to two arbitrary functions.

The observational past null cone is specified in the form, $t = \hat{t}(r)$. We denote the areal radius R on $t = \hat{t}(r)$ by \mathcal{R} . Then by Eq.(4), we can regard \dot{R} on $t = \hat{t}(r)$ as a function of \mathcal{R} , E and r ;

$$\dot{R}(\hat{t}(r), r) = \mathcal{R}_1(\mathcal{R}, E, r) \equiv \sqrt{\frac{2M_0 r^3}{\mathcal{R}} + 2Er^2}, \quad (9)$$

By differentiating Eqs.(4) and (6), R' and \dot{R}' on $t = \hat{t}(r)$ can be expressed as functions of \mathcal{R} , \hat{t} , E , E' , t_B , t'_B and r ;^{*}

$$\begin{aligned} R'(\hat{t}(r), r) &= \mathcal{R}_2(\mathcal{R}, \hat{t}, E, E', t_B, t'_B, r) \\ &\equiv -\left(\mathcal{R} - \frac{3}{2}[\hat{t} - t_B]\mathcal{R}_1\right) \frac{E'}{E} - \mathcal{R}_1 t'_B + \frac{\mathcal{R}}{r}, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \dot{R}'(\hat{t}(r), r) &= \mathcal{R}_3(\mathcal{R}, \hat{t}, E, E', t_B, t'_B, r) \\ &\equiv \frac{1}{2} \left(\mathcal{R}_1 - 3 \frac{M_0 r^3}{\mathcal{R}^2} [\hat{t} - t_B] \right) \frac{E'}{E} \\ &\quad + \frac{M_0 r^3}{\mathcal{R}^2} t'_B + \frac{\mathcal{R}_1}{r}. \end{aligned} \quad (11)$$

^{*}If $E(r) = 0$, we should omit the terms proportional to E' in Eqs. (10) and (11).

The observational past null cone $t = \hat{t}(r)$ satisfies

$$\frac{d\hat{t}}{dr} = -\frac{\mathcal{R}_2(\mathcal{R}, \hat{t}, E, E', t_B, t'_B, r)}{\sqrt{1 + 2Er^2}}. \quad (12)$$

The redshift $z(r)$ along the past null cone is given by

$$\frac{dz}{dr} = \frac{1 + z}{\sqrt{1 + 2Er^2}} \mathcal{R}_3(\mathcal{R}, \hat{t}, E, E', t_B, t'_B, r). \quad (13)$$

The total derivative of \mathcal{R} on the past null cone is written as

$$\frac{d\mathcal{R}}{dr} = \left(1 - \frac{\mathcal{R}_1(\mathcal{R}, E, r)}{\sqrt{1 + 2Er^2}}\right) \mathcal{R}_2(\mathcal{R}, \hat{t}, E, E', t_B, t'_B, r). \quad (14)$$

Our basic equations are Eqs.(12)–(14). These three equations can be regarded as a system of first order ordinary differential equations for three of the five functions $\mathcal{R}(r)$, $\hat{t}(r)$, $E(r)$, $t_B(r)$ and $z(r)$. In order to integrate these equations, we should specify two conditions on these five functions. The luminosity distance $D_L(z)$ is related to \mathcal{R} [15] as

$$\mathcal{R} = \frac{D_L(z)}{(1 + z)^2}. \quad (15)$$

As already mentioned, we assume that $D_L(z)$ is given by Eq.(1). Further we will specify one condition for E , t_B or the combination of them.

At first we consider pure Big-Bang time inhomogeneity. In this case the curvature function $E(r)$ is set to be constant. From Eqs. (13) and (14), we have equations for $z(r)$ and the Big-Bang time function $t_B(r)$. The model is specified by $\Omega_0 \equiv 2M_0/H_0^2$ which is the present central density $3M_0/4\pi$ divided by the present central critical density $\rho_{\text{crit}} = 3H_0^2/8\pi$, where H_0 is the present central Hubble parameter and we set it to be unity. We numerically integrate these two differential equations from $r = 0$ for ten Ω_0 from 0.1 to 1.0. The initial conditions are given by $z = 0$ and $t_B = 0$.

From Eq. (4) $R' > 0$ for positive density while from Eq. (9) $\dot{R}' > 0$ for monotonically increasing $z(r)$ so that the integration is terminated either of the following inequalities is violated,

$$R' > 0 \quad \text{or} \quad \dot{R}' > 0. \quad (16)$$

In Fig. 1 we show the relation between the parameter Ω_0 and the redshift when the integration is terminated. For low $\Omega_0 = 0.1 - 0.4$ (open triangles), shell-crossing singularities occur when $d\mathcal{R}/dz = 0$. For high $\Omega_0 = 0.5 - 1.0$ (open square), the second condition of Eq. (16) is violated first. This occurs when $\mathcal{R} = 2M$.

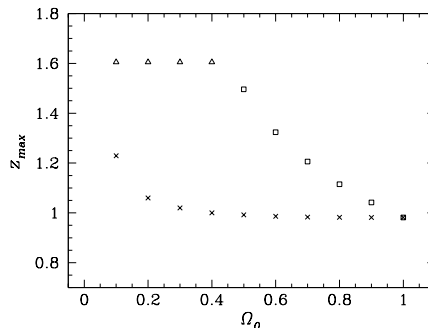


FIG. 1. Plots of maximum redshifts when either of inequalities in Eq. (16) is violated as a function of the present density parameter. The open triangles and the open squares are the ones for the Big-Bang time inhomogeneity. The cross marks are the ones for the curvature inhomogeneity case.

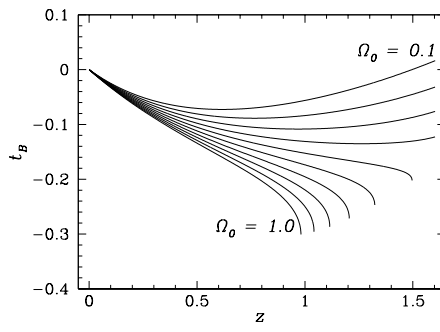


FIG. 2. Plots of the Big-Bang time functions as a function of the redshift z .

Fig. 2 shows the Big-Bang time functions t_B for each Ω_0 . For all Ω_0 the Big-Bang time functions t_B decrease as z increases up to $z \sim 0.5$. This result is related to the fact that the expansion of our universe looks to be accelerative up to $z \sim 0.5$. In inhomogeneous models, the apparent acceleration is realized if the recession velocity of mass shells does not increase so steeply along the observational past null cone as the case of the homogeneous and isotropic universe filled with dust. To construct such a situation in our model, we need to prepare older shell, i.e., more decelerated shell by gravity, for more distant one on the past null cone. This is the reason why the function t_B decreases.

In Figs. 3 we plot the redshift space density

$$\hat{\rho}(z) = \rho \frac{4\pi R^2 R' dr}{4\pi z^2 dz} = \Omega_0 \frac{r^2}{z^2} \frac{dr}{dz} \rho_{\text{crit}}, \quad (17)$$

along the past null cone. Observations of the mass distribution along the past null cone would give us this density profile.

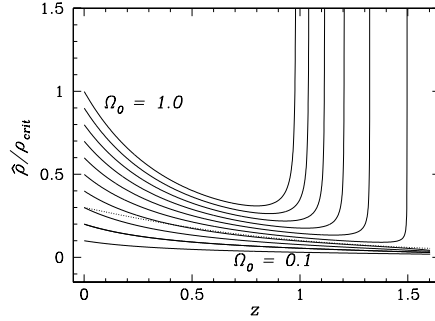


FIG. 3. Plots of the redshift space density $\hat{\rho}$ divided by the central critical density. The dotted line denotes the $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ homogeneous model.

Next we consider the pure curvature inhomogeneity. In this case the Big-Bang time function $t_B(r)$ is set to be zero. From Eqs. (12), (13) and (14) we obtain three differential equations for three variables $z(r)$, $E(r)$ and $\hat{t}(r)$. We numerically integrate these three differential equations from $r = 0$. The initial conditions are given by $z = 0$, $E = (1 - \Omega_0)/2$ and

$$\hat{t}(0) = \frac{\Omega_0}{2} \frac{(\sinh \eta_0 - \eta_0)}{(1 - \Omega_0)^{\frac{3}{2}}}, \quad (18)$$

where

$$\eta_0 = \ln \left(\frac{2 - \Omega_0}{\Omega_0} + \sqrt{\left(\frac{2 - \Omega_0}{\Omega_0} \right)^2 - 1} \right). \quad (19)$$

The present central cosmological time $\hat{t}(0)$ and η_0 are obtained from Eq. (6).

In Fig. 1 we show the relation between the parameter Ω_0 and the redshift when the integration is terminated (cross marks). For the case of curvature inhomogeneity, it was shown that the second condition of Eq. (16) is violated first [16].

Fig. 4 shows the curvature functions E for each Ω_0 . We can see E decreases as z increases except for the $\Omega_0 = 1.0$ case.

The decreasing E is consistent with the apparent acceleration. E determines the specific energy of the dust elements so that the “initial” velocity is slower for more distant shells. This causes the apparent acceleration since the velocity at $r = 0$ can be the largest.

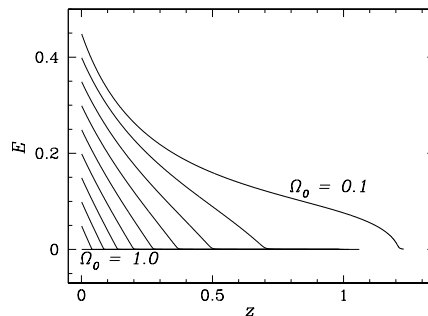


FIG. 4. Plots of the curvature functions.

Fig. 5 shows the redshift space density $\hat{\rho}$ along the past null cone as a function of z .

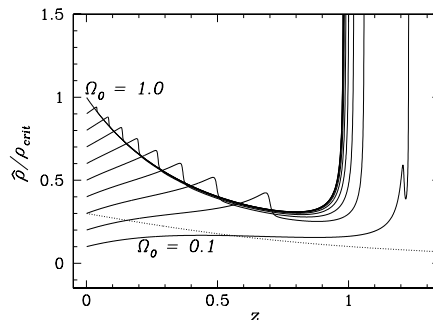


FIG. 5. Plots of the redshift space density divided by the present central critical density. The dotted line denotes the $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ homogeneous model.

In this letter we have constructed inhomogeneous dust models without the dark energy. These models can be consistent with the observed $D_L(z)$ up to $z = 1$ since from Fig. 1 we have no difficulties up to $z \sim 1$ for any parameters in both the Big-Bang time inhomogeneity and the curvature inhomogeneity cases. For $z > 1$, we have difficulties in our inhomogeneous dust models. Recently, the SNIa at the redshift of ~ 1.7 was found [17,18] with rather large error bars. However only a single SNIa at the redshift of ~ 1.7 is not enough to construct the accurate $D_L(z)$ although it seems to rule out the ‘grey-dust’ hypothesis. If future observations confirm $D_L(z)$ up to $z \sim 2$ with $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$, our inhomogeneous dust models are incompatible with the observations and some form of the dark energy will be the case. However, if future observations confirm that $D_L(z)$ for $z > 1$ does not follow Eq. (1) appreciably, the possibility of our inhomogeneous dust models remain to be studied more extensively. In such a case, the first Doppler peak as well as the higher ones will give us another constraints to the inhomogeneous universe models.

One may suspect that the existing observations in $0 < z < 1$ such as : (i) evolution of cluster abundance, (ii) lensing rate, (iii) ages of stellar populations already rule out the inhomogeneous models.

Using the cluster temperature evolution data for $0.3 < z < 0.8$, it was reported that the best-fit value of $\Omega_m = 0.45 \pm 0.1$ for open universe and $\Omega_m = 0.3 \pm 0.1$ for flat universe [19]. However recent analysis shows that the systematic error is comparable to the statistical error [20]. So we may say that $0.1 < \Omega_m < 0.5$ for $0.3 < z < 0.8$ data. It is not clear that the Press-Schechter formalism can be applied to our inhomogeneous models. One of the possible estimate

would be based on the locally homogeneous approximation. As we know, the massive cluster evolution is very sensitive to the matter density. It seems that the model with local density parameter Ω_m which largely conflicts with the best-fit value would not explain the observed cluster evolutions. The pure curvature inhomogeneity case with $\Omega_0 \gtrsim 0.2$ may be difficult to survive because it is approximated by flat universe at high z . Also the Bang time inhomogeneity case with $\Omega_0 \sim 1.0$ can not survive. However, it can be expected that the pure Bang time inhomogeneity with $\Omega_0 \sim 0.5$ and the pure curvature inhomogeneity with $\Omega_0 \sim 0.1$ would predict the observed cluster abundances.

The estimate of the lensing rate and the distribution of the separation of the images depend on the model of the mass distribution of the lensing object and the luminosity function of the source objects as well as the cosmological parameters. However it has been shown that the dependence on the lens model and parameters is much larger than that on the cosmological parameters [21,22]. In addition, the mass distribution of the lensing objects would deeply depend on baryon density Ω_b [23]. Therefore we conclude that the estimate of the cosmological parameters from the lensing rate and the distribution of the separation of the images is difficult at present so that we can not rule out the inhomogeneous model.

As shown in Fig. 6, the look back times along the past null cone have little difference between the inhomogeneous model and the corresponding homogeneous model with cosmological constant for $z < 0.5$. For $z \sim 1$, the difference appears, but some of the inhomogeneous models are not so different from the homogeneous model even there. The ages of stellar population would not distinguish the inhomogeneous model from the homogeneous one.

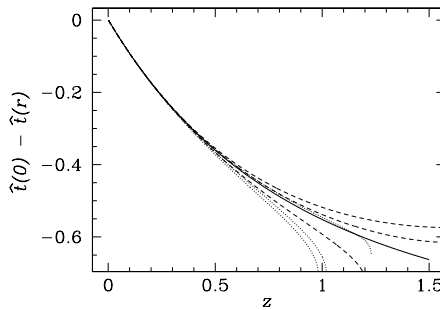


FIG. 6. Plots of the look back time along the past null cone. Solid line denotes the homogeneous $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ case. Broken and dotted lines denote the pure Big-Bang time and the pure curvature inhomogeneity case of $\Omega_0 = 0.1, 0.3, 0.7$ in descending order, respectively.

As a result, the model dependence including various undetermined parameters and the observational uncertainty are much larger than the dependence on the cosmological parameters. Therefore we think that these observations can not easily rule out the inhomogeneous model.

Here we comment on how we can be place away from the center of the symmetry. The displacement from the center would correspond to the dipole mode of CMB. Therefore we can be ~ 10 Mpc away from the center.

In conclusion, the dark energy is not the only solution to the apparent acceleration of the present universe but inhomogeneous dust models can also explain the observations.

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